NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3221

STUDY OF THE SUBSONIC FORCES AND MOMENTS ON AN INCLINED PLATE OF INFINITE SPAN

By Bradford H. Wick

Ames Aeronautical Laboratory
Moffett Field, Calif.

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SUMMARY

A study has been made of existing experimental and theoretical results for an inclined flat plate of infinite span, and of the extent to which the results are indicative of those for thin airfoil sections. The study included an examination of the flow about an inclined plate, the forces on the plate, and the adequacy of theory in predicting the forces. Theories considered were the well-known thin-airfoil theory, and the theory of discontinuous potential flow and modifications thereof. The effects of compressibility were examined.

The results of the study indicate that there are two important ranges of angle of attack differing by the extent of flow separation on the upper surface. At angles of attack below about 8°, flow separation and reattachment occur, and the well-known thin-airfoil theory is adequate for predicting the lift and normal force on the plate. Similar results were noted for thin airfoil sections. At the higher angles of attack the flow is completely separated from the upper surface as is assumed in the discontinuous potential-flow theory for an inclined flat plate. The theory, however, is entirely inadequate. A simple empirical modification of the theory is suggested; the modified theory provides a good first approximation of the forces and moments on thin airfoil sections with the flow completely separated from the upper surface. Effects of compressibility were evident from the available experimental data; however, the effects were not defined sufficiently for evaluating methods of prediction.

INTRODUCTION

The results of studies, by early researchers in hydrodynamics, of the flow about and the resultant forces on an inclined flat plate of infinite span, heretofore, have had little practical application. The type of flow considered, consisting of detached flow over the upper surface (i.e., rearward surface) and attached flow over the lower surface, was not encountered on conventional airfoils in the angle-of-attack range of practical interest. With the introduction of thin airfoils and, in particular, those with sharp leading edges, the foregoing circumstance no longer exists. The separated type of flow has been found to occur on thin unswept wings at and above the angle of attack for maximum lift, on thin sweptback wings considerably prior to wing maximum lift, and on thin propellers when operating at take-off conditions. It appeared worthwhile, therefore, to make a study of existing theoretical and experimental results for the flat plate and to determine their applicability to thin airfoil sections. The results of the study are reported herein.

NOTATION

c_d section drag coefficient, $\frac{\text{drag}}{q_o c}$

c_l section lift coefficient, $\frac{\text{lift}}{q_o c}$

$c_{m_{c/4}}$ section pitching-moment coefficient, moment center at $c/4$, $\frac{\text{pitching moment}}{q_o c^2}$

c_n section normal-force coefficient, $\frac{\text{normal force}}{q_o c}$

P pressure coefficient, $\frac{p-p_o}{q_o}$

$P_{u_{av}}$ average upper-surface pressure coefficient

$P_{l_{av}}$ average lower-surface pressure coefficient

c chord

M Mach number of free stream

p local static pressure

p_o free-stream static pressure

q_o free-stream dynamic pressure

V local velocity

V_o free-stream velocity

x_{cp} center-of-pressure location, distance along chord from leading edge, fractions of chord length

α angle of attack of chord plane, deg

RESULTS AND DISCUSSION OF STUDY

There are two theories which are pertinent to an inclined flat plate of infinite span. One is the so-called discontinuous potential-flow theory (ref. 1, pp. 330-336) which treats the case where the flow is completely detached from the upper surface; the other is the well-known thin-airfoil theory (ref. 1, pp. 24-53) which treats the case of unseparated flow. Since the former theory has been of little practical interest and, consequently, is not so well known, the following brief discussion is believed in order.

The first complete treatment of the separated type of flow, using methods of classic hydrodynamics appears to be that presented by Rayleigh in 1876. He treated both the case of the plate oblique to the stream and normal to the stream. Kirchhoff some years earlier (in 1869) had considered both cases but presented calculated results only in the case of the plate normal to the stream. Although working independently, their approach was a common one, making use of Helmholtz's hypothesis of a surface of discontinuity (i.e., a surface which separates two streams of different velocities). As a consequence of the use of this hypothesis, their approach is known in the literature as the method of discontinuous potential flow.

A complete description of the method is given in reference 1. The salient features of the method are as follows: It is assumed that lines of discontinuity start at the leading and trailing edges of the plate and extend to infinity. (See fig. 1.) Within the two lines the fluid is assumed to be at rest with respect to the plate. Outside these lines the flow is assumed to be smooth and steady. As a result of the flow conditions assumed, the pressure in the wake (i.e., the region bounded by the lines of discontinuity) is constant and equal to the free-stream static pressure, and the velocity outside the wake is equal to the free-stream velocity.

The solution for the force on the plate due to the described flow about the plate is, in coefficient form,

$$c_n = \frac{2\pi \sin \alpha}{4 + \pi \sin \alpha} \tag{1}$$

The center-of-pressure location in fractions of the chord from the leading edge is

$$x_{cp} = 0.50 - 0.75 \frac{\cos \alpha}{4 + \pi \sin \alpha} \tag{2}$$

The derivation of the equation for c_n is given in references 1 and 2; the equation for the center-of-pressure location is from the derivation given in reference 2, wherein the location is referred to the midchord rather than the leading edge.

Since there is only a normal force acting on the plate, the equations for the coefficients of lift and drag are

$$c_l = \frac{2\pi \sin \alpha \cos \alpha}{4 + \pi \sin \alpha} \tag{3}$$

$$c_d = \frac{2\pi \sin^2 \alpha}{4 + \pi \sin \alpha} \tag{4}$$

The foregoing equations are presented in graphical form in figure 2, together with the thin-airfoil-theory results and the experimental results for a flat plate as measured by Fage and Johansen (ref. 3). (The experimental results are uncorrected for the constraint of the tunnel walls. It is stated in the reference report that the measured values of the normal-force coefficient should be reduced by amounts varying from 8 percent at $\alpha = 30^{\circ}$ to 13.5 percent at $\alpha = 90^{\circ}$.) The adequacy of thin-airfoil theory in accounting for the magnitude of the normal force and the lift on the plate in the low angle-of-attack range, and the inadequacy of the Rayleigh-Kirchhoff theory throughout the entire angle-of-attack range are readily apparent from the figure. In the case of drag coefficient and center-of-pressure location, both theories are inadequate throughout the angle-of-attack range.

That thin-airfoil theory would be applicable in predicting the lift of a flat plate at low angles of attack may seem surprising in view of the probable separation of flow from the leading edge of the plate. It appears, however, from theoretical considerations and an examination of the lift and flow measurements on a thin sharp-edge airfoil section (ref. 4), that the applicability of thin-airfoil theory is determined primarily by the flow condition at the trailing edge. The lift measurements as given in reference 4 for the thin sharp-edge airfoil section are reproduced in figure 3; the data were not corrected for tunnel-wall effects. Also shown are the values of lift indicated by thin-airfoil theory and the Rayleigh-Kirchhoff theory. The extent of the separated-flow region is indicated in figure 4, which is a reproduction of a figure in reference 4. The boundary of the separated-flow region was defined by the zero-velocity point in velocity distributions above the surface which were determined by rakes of conventional static- and total-pressure tubes. It is noted from figure 3 that, as for the flat plate, the lift variation with angle of attack was essentially that specified by thin-airfoil theory up to about 7.5°, and then deviated rapidly. The data on the extent of flow separation (fig. 4) show that the flow separated from the leading edge at a very small angle of attack and then reattached farther back along the surface. The point of reattachment moved farther back with increasing angle of attack until at 7.5°, the angle of the lift-curve divergence, the flow was completely separated from the upper surface. That the amount of lift developed is primarily dependent upon the flow at the trailing edge is, of course, to be expected, since in thin-airfoil theory the amount of lift is established by satisfying the Kutta condition at the trailing edge. Leading-edge flow separation could have an effect on the amount of lift developed, however, through a change in the boundary-layer thickness at the trailing edge. Another way that the leading-edge flow separation could possibly influence the amount of lift is that it produces, in effect, a cambered airfoil formed by the plate and the separated-flow region. If such were the case, the thin-airfoil-theory solution for lift due to angle of attack might not be expected to be applicable. However, in view of the lift results obtained, it is apparent that leading-edge separation had little effect on the circulation at a given angle of attack as long as the flow reattached to the surface well ahead of the trailing edge.

With complete detachment of the upper-surface flow, a flow condition assumed in the Rayleigh-Kirchhoff theory is satisfied, but, as was noted, the theory fails to define the forces and moments on the plate. It has been fairly well established that the failure is due to differences between the assumed and actual wake conditions. As noted in reference 1, flow observations have shown the fluid behind the plate to have a definite vortical motion rather than being at rest as assumed in the theory. Further, the wake boundaries are actually vortex sheets rather than surfaces of discontinuities as assumed in the theoretical treatment. (See reference 5 for the results of a detailed study of the structure of the sheets.) Due to the presence of the vortices in the wake, a pressure lower than that of the free stream is developed at the upper surface of the plate. How much the pressure differs from that of the free stream is indicated in the following table. Also shown are the experimental values of the average lower-surface pressure coefficient and the theoretical values for both surfaces. The experimental values are from reference 3 and have been corrected for wind-tunnel-wall effects. (See the appendix for the method of correction.)

α, deg	$P_{u_{av}}$		$P_{l_{av}}$	
	Experimental	Theoretical	Experimental	Theoretical
15	-0.58	0	0.25	0.34
30	-.80	0	.41	.56
40	-.90	0	.53	.67
50	-.98	0	.62	.75
60	-1.04	0	.69	.81
70	-1.04	0	.75	.85
80	-1.05	0	.78	.87
90	-1.05	0	.79	.88

It can be seen from the table that the differences between experiment and theory are large in the case of the upper surface and relatively small in the case of the lower surface. The difference between the experimental and the theoretical values of the upper-surface pressure coefficient varies from about 60 to 70 percent of the corresponding experimental normal-force coefficient, whereas for the lower surface the difference varies from about 5 to 12 percent. Efforts to improve the Rayleigh-Kirchhoff theory obviously should be and have been directed toward obtaining a method of predicting the wake conditions and their effect on the upper-surface pressure.

The only existing modification known is that proposed by D. Riabouchinsky. His proposal is briefly described in reference 1. It is stated therein that he suggested an assumption of a second plate downstream and the calculation of the shape of the wake between the two plates, the size and location of the second plate being chosen in such a way that the pressure in the wake was equal to the value found experimentally. Thus Riabouchinsky's method is essentially empirical. A simpler empirical approach is suggested in the following section of the report.

Empirical Modification of the Rayleigh-Kirchhoff Theory

Since the Rayleigh-Kirchhoff theory adequately accounts for the average pressure over the lower surface of a plate, a simple empirical modification of the theory would be to substitute experimental values of the upper-surface pressure coefficient directly in place of the theoretical. The only values found to be available for a flat plate were those measured by Fage and Johansen (ref. 3) and given in the preceding table. A comparison of these values with those available for airfoil sections at high angles of attack indicated the desirability of obtaining additional values for a flat plate. In order to provide additional values, measurements were made of the average pressure over the upper surface of a 2-inch-chord plate in a wind tunnel with a 2- by 5-foot test section; the plate spanned the 2-foot dimension of the test section. The resulting values of $P_{u_{av}}$, corrected for tunnel-wall effects by the method given in the appendix, are presented in figure 5 along with the flat-plate values from reference 3. Also shown in figure 5 are the values for several airfoil sections with completely detached upper-surface flow. The values for the NACA 0015 section were obtained from tests of the section through an angle-of-attack range of 0° to 180° (ref. 6); corrections for tunnel-wall effects were not required (see Appendix II of ref. 6). The values for the 64A-series section were obtained from tests of the sections at angles of attack up to 28° at a Mach number of approximately 0.3, and include tunnel-wall corrections by the method given in the appendix of the present report; the Mach number is about 0.2 higher than the Mach numbers of the tests of the plates and the NACA 0015 section. (The effect of the difference is small and has been approximately accounted for by using the theoretical compressibility factors discussed later in the report.)

The flat-plate values of the present report and the values for the various airfoil sections were used in establishing the curve shown in figure 5. It is believed that this curve provides a reasonably good definition of values of $P_{u_{av}}$ to use in the modification of the Rayleigh-Kirchhoff theory. Although the curve is based on data covering only a Reynolds number range of 0.15 to 1.23 million, the curve should be applicable to higher Reynolds number.

Using the values of $P_{u_{av}}$ from the faired curve of figure 5, to modify the Rayleigh-Kirchhoff theory, the force and moment coefficients are given by the following equations:

$$c_n = \frac{2\pi \sin \alpha}{4 + \pi \sin \alpha} - P_{u_{av}} \tag{5}$$

$$c_l = \left(\frac{2\pi \sin \alpha}{4 + \pi \sin \alpha} - P_{u_{av}}\right) \cos \alpha \tag{6}$$

$$c_d = \left(\frac{2\pi \sin \alpha}{4 + \pi \sin \alpha} - P_{u_{av}}\right) \sin \alpha \tag{7}$$

$$c_{m_{c/4}} = \frac{2\pi \sin \alpha}{4 + \pi \sin \alpha}(0.25 - x_{cp}) + \frac{P_{u_{av}}}{4} \tag{8}$$

where x_{cp} is given by equation (2). The results given by these equations are in good agreement with the flat-plate data (ref. 3) corrected for tunnel-wall effects.

In order to indicate the degree of applicability of the modified flat-plate theory to thin airfoil sections, the coefficient values given by the foregoing equations are compared in figure 6 with corresponding measured values for several thin airfoil sections (refs. 7 and 8); also shown in the figure are thin-airfoil-theory values. (Although the values of $P_{u_{av}}$ to be used in equations (5) through (8) were established from data for both airfoil sections and plates, the equations are strictly applicable only to a plate or airfoil section with a flat lower surface, since the Rayleigh-Kirchhoff theory applies only to a flat lower surface.) The indication of applicability is limited somewhat by the angle-of-attack range and scatter of the experimental values. For the angle-of-attack range covered, however, it is concluded that the modified Rayleigh-Kirchhoff theory provides a good first approximation of the forces and moments on thin airfoil sections with completely detached upper-surface flow.

A brief examination has been made of the effects of compressibility on the separated (i.e., discontinuous) type of flow considered herein. The compressible-flow counterpart of the Rayleigh-Kirchhoff theory was given by Chaplygin in 1902 (ref. 9). His solution can be applied approximately as a compressibility factor in a manner analogous to that used in applying the well-known Prandtl-Glauert relation. The factor from Chaplygin's solution is approximately $1/[1 - (0.5M)^2]$. A considerably smaller compressibility effect is indicated by Chaplygin's solution than would be indicated by the Prandtl-Glauert relation. It may seem questionable to consider the use of the Prandtl-Glauert relation in this case, since it is normally associated with the continuous type of steady potential flow. There appears to be no reason, however, why it should be invalid because of the discontinuity in the flow (fig. 1) assumed in the Rayleigh-Kirchhoff theory, since the theoretical force is due to the continuous steady potential flow occurring outside of the area bounded by the plate and wake. In the case of the actual flow and force on a plate, there is no theoretical basis for applying either the Chaplygin solution or the Prandtl-Glauert relation because of the previously discussed lack of a theoretical treatment of the large wake effect. It appears of interest, nevertheless, to examine their applicability in the light of available experimental evidence. Values of lift coefficient predicted by applying either the Chaplygin compressibility factor, or the Prandtl-Glauert relation to the modified Rayleigh-Kirchhoff theory are compared in figure 7 with measured values for three 6-percent-thick airfoil sections. (The experimental data, from references 7 and 8, were recorrected for tunnel-wall effects by the method given in the appendix of the present report.) Due to unaccountable differences and scatter in the available data, no definite conclusion can be reached. Applicability of the Prandtl-Glauert relation is generally indicated by the data for the NACA 64-006 section, and the Chaplygin solution by the data for the other two sections.

CONCLUDING REMARKS

The study of existing experimental and theoretical results for an inclined flat plate of infinite span revealed the following facts regarding the types of flow occurring about the plate, and the adequacy of theory in predicting the forces on the plate. At low angles of attack, below about 8°, flow separation and reattachment occurs on the upper surface, and for this angle range thin-airfoil theory is adequate for predicting the lift and normal force on the plate. At higher angles of attack the flow is completely separated from the upper surface, a condition which is assumed in the Rayleigh-Kirchhoff theory for an inclined plate. The Rayleigh-Kirchhoff theory, however, is entirely inadequate for predicting the magnitude of the lift and normal force on the plate with complete detachment of the upper-surface flow.

The deficiency of the Rayleigh-Kirchhoff theory is due to differences between assumed and actual wake conditions; as a consequence, the average upper-surface pressure given by theory is considerably different from experimental values. A simple empirical modification of the Rayleigh-Kirchhoff theory that appears promising is to substitute experimentally determined values of the upper-surface pressure in place of the theoretical. Comparison of values of lift, normal-force, drag, and pitching-moment coefficient given by the modified theory with values measured for thin round-nose airfoil sections indicates that the modified theory provides a good first approximation of the forces and moments on such airfoil sections when the flow is completely separated from the upper surface. Experimental data indicate an effect of compressibility on the lift of airfoil sections with completely detached upper-surface flow; the effect of compressibility was not sufficiently defined, however, for methods of prediction to be evaluated.

Ames Aeronautical Laboratory
National Advisory Committee for Aeronautics
Moffett Field, Calif., May 4, 1954

APPENDIX

TUNNEL-WALL CORRECTIONS FOR AN INCLINED FLAT PLATE OF INFINITE SPAN

The method of correction is a simple extension of the method given in reference 10 for correcting the drag of an infinite-span plate inclined 90° to the stream in a closed tunnel. It is shown in reference 10 that the effect of the walls can be treated as simple wake blockage. It was empirically established that the area blocked is equal to the area of the plate. The equivalent free-air velocity is thus

$$V_o = V_o' \left[1 + \frac{c/h}{1-(c/h)}\right]$$

where

V_o equivalent free-air velocity

V_o' tunnel velocity

c chord length of plate

h dimension of tunnel cross section normal to plate span

To extend this approach to angles of attack other than 90°, it is assumed that the wall effects can still be treated as simple blockage and that the blocked area is equal to the frontal area of the plate. (The fact that this reduction in area does not occur at one streamwise position is neglected.) It is also assumed that the approach is applicable to compressible subsonic flow. The resulting equations for the velocity, Mach number, and dynamic pressure are

$$\frac{V_o}{V_o'} = 1 + \frac{K}{1-(M')^2}$$

$$\frac{M}{M'} = 1 + \frac{1 + 0.2(M')^2}{1 - (M')^2} K$$

$$\frac{q_o}{q_o'} = 1 + \frac{2 - (M')^2}{1 - (M')^2} K$$

where

$$K = \frac{(c/h) \sin \alpha}{1 - (c/h) \sin \alpha}$$

c and h are as previously defined, and the primed symbols are the uncorrected values. The ratios of corrected to uncorrected values of the lift, normal-force, drag, and pitching-moment coefficient are equal to the reciprocal of the corresponding values of q_o/q_o'; for example

$$\frac{c_l}{c_l'} = \frac{q_o'}{q_o}$$

The corrected value of the pressure coefficient is

$$P = \frac{\dfrac{2K}{1-(M')^2} + P'}{q_o/q_o'}$$

It is to be noted that the foregoing method of correction neglects any possible effects of the tunnel walls on the angle of attack or the center of pressure. It is believed, however, that such effects are small.

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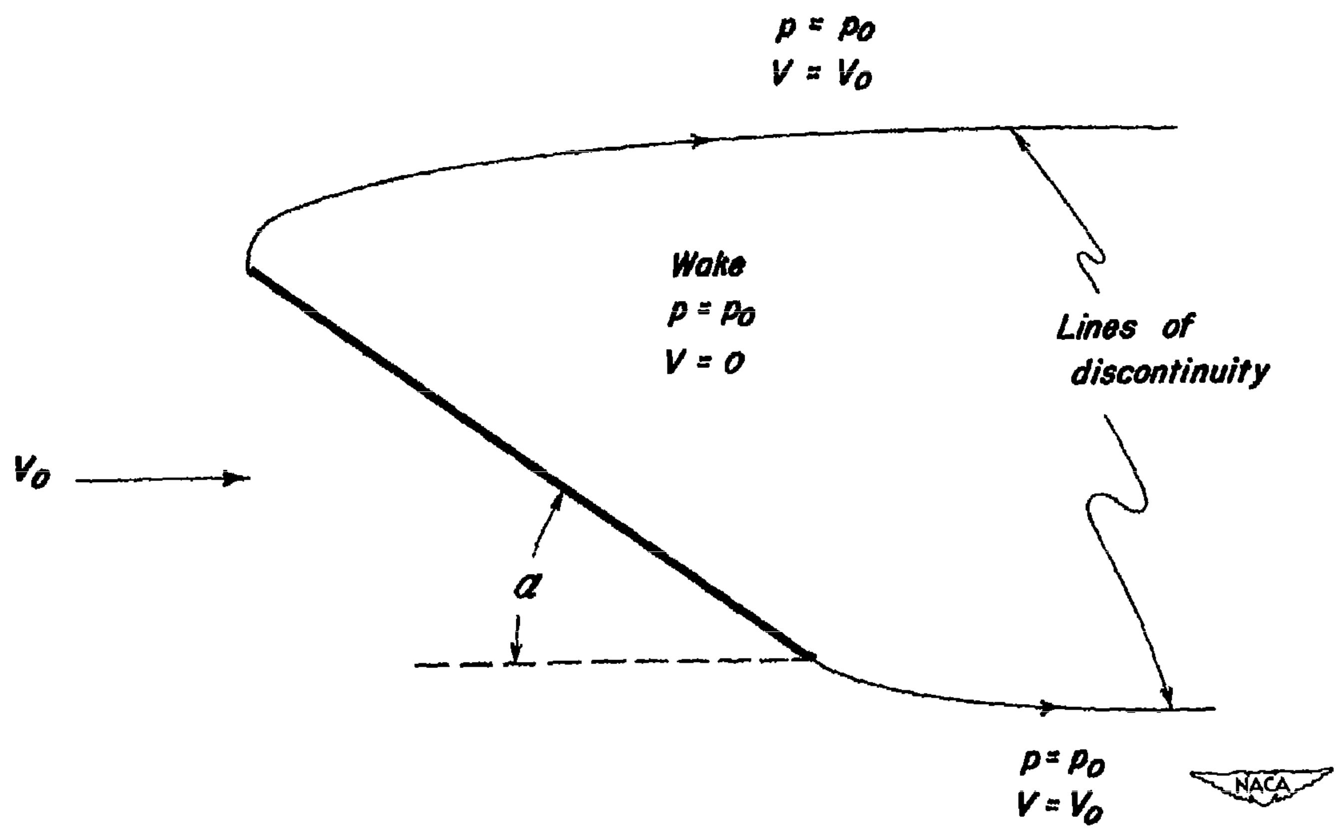
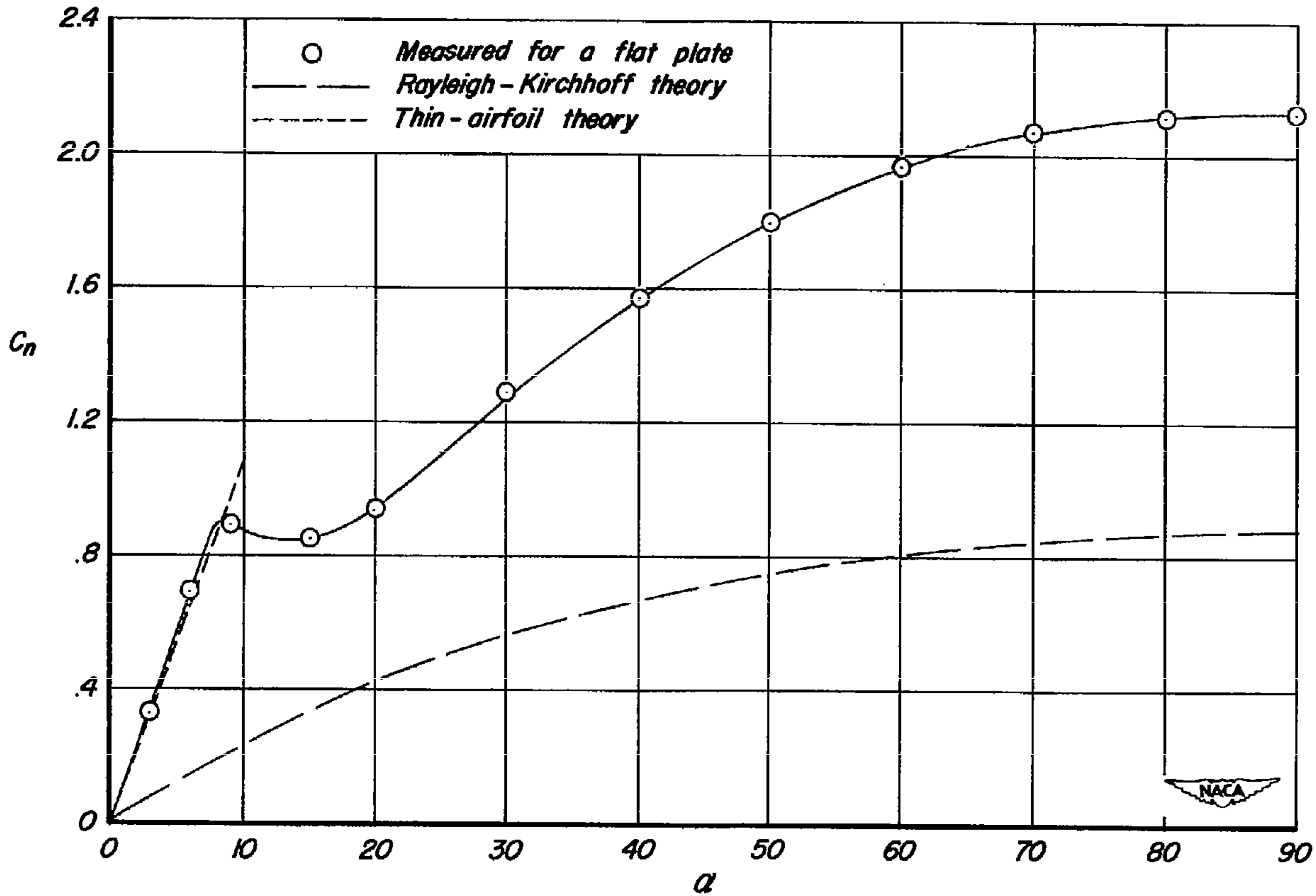
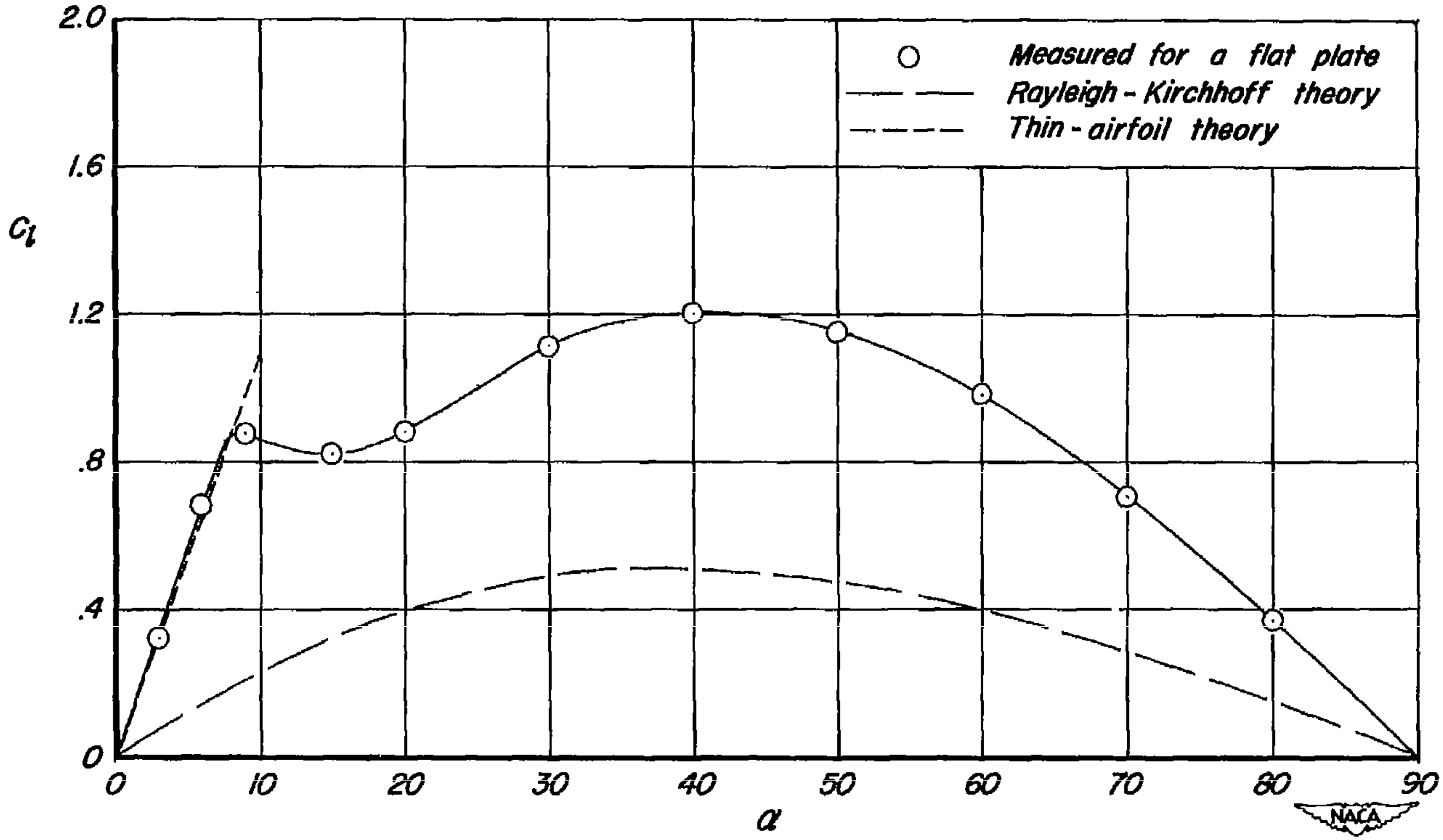


Figure 1.- Flow about an inclined flat plate of infinite span, as assumed in the Rayleigh-Kirchhoff theory.



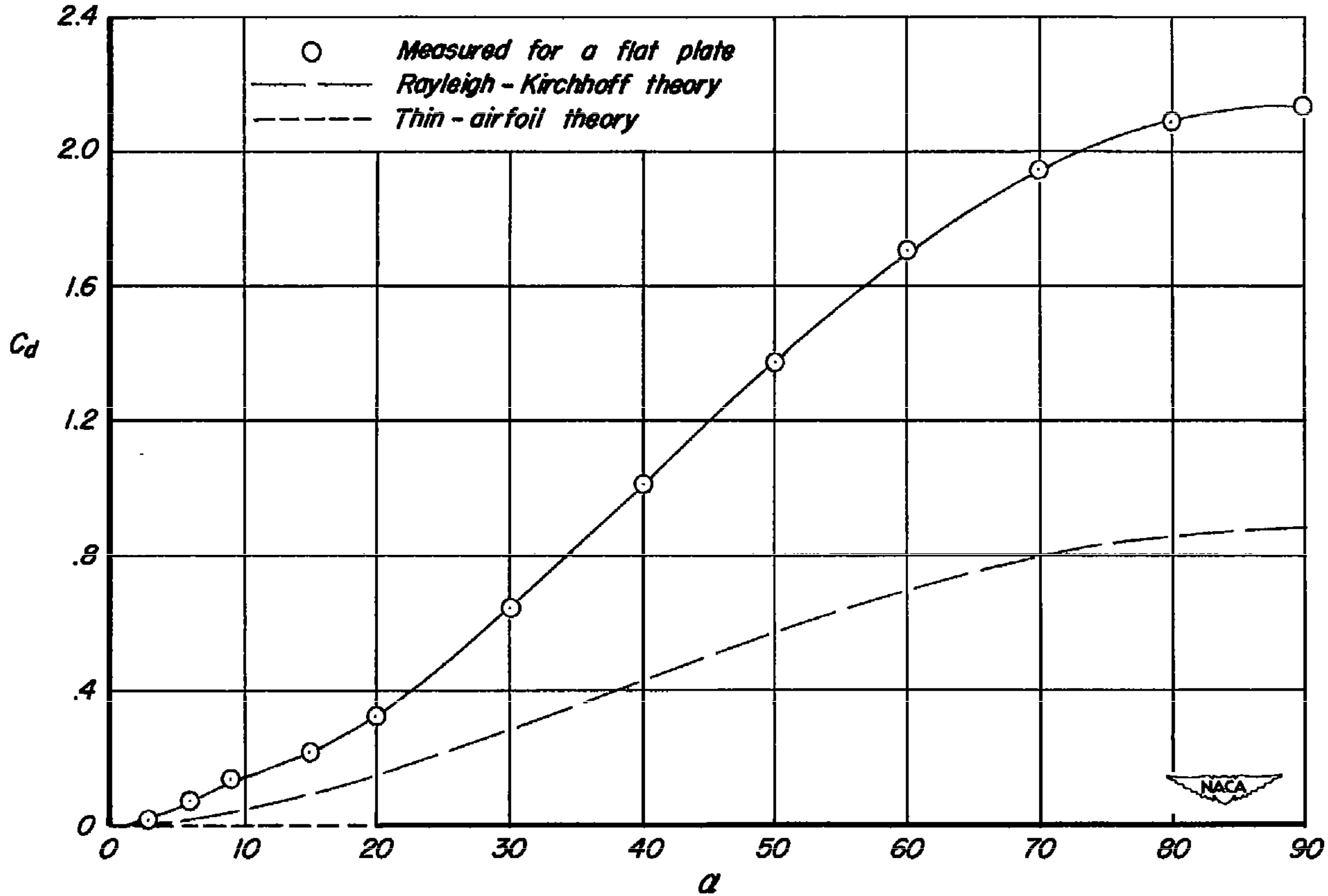
(a) c_n vs. α

Figure 2.- Comparison of the measured characteristics of a flat plate (ref. 3) with the characteristics given by the Rayleigh-Kirchhoff theory and by thin-airfoil theory.



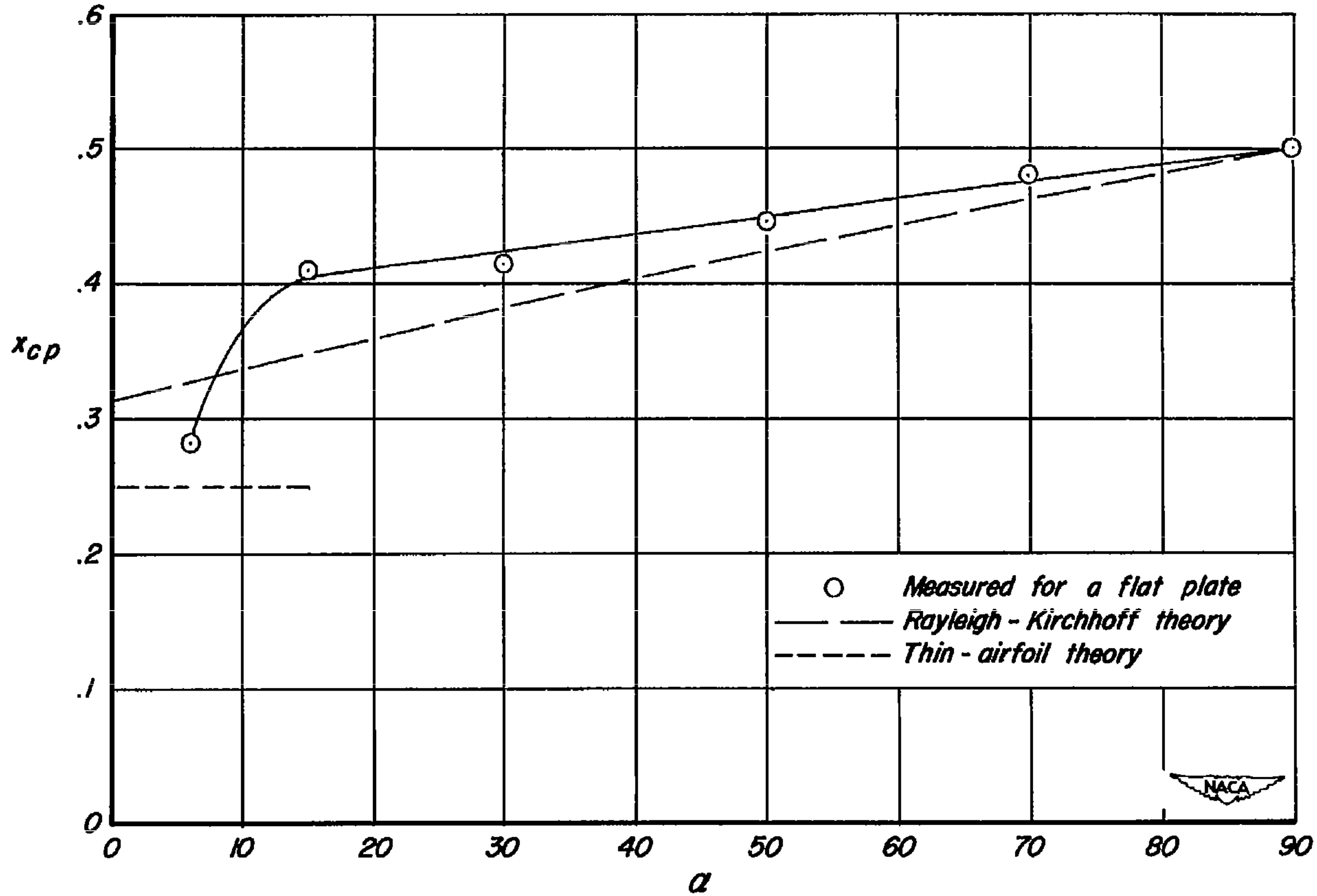
(b) c_l vs. α

Figure 2.- Continued.



(c) c_d vs. α

Figure 2.- Continued.



(d) x_{cp} vs. α

Figure 2.- Concluded.

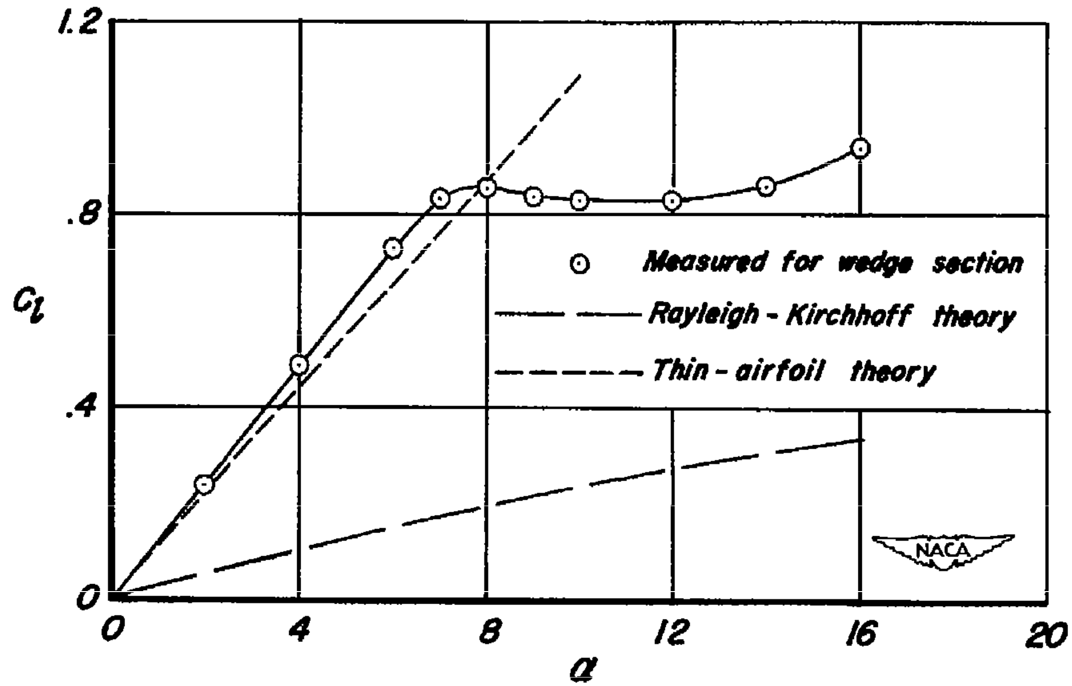


Figure 3.- Comparison of the measured lift characteristics of a faired double-wedge airfoil section (ref. 4) with lift characteristics given by the Rayleigh-Kirchhoff theory and by thin-airfoil theory.

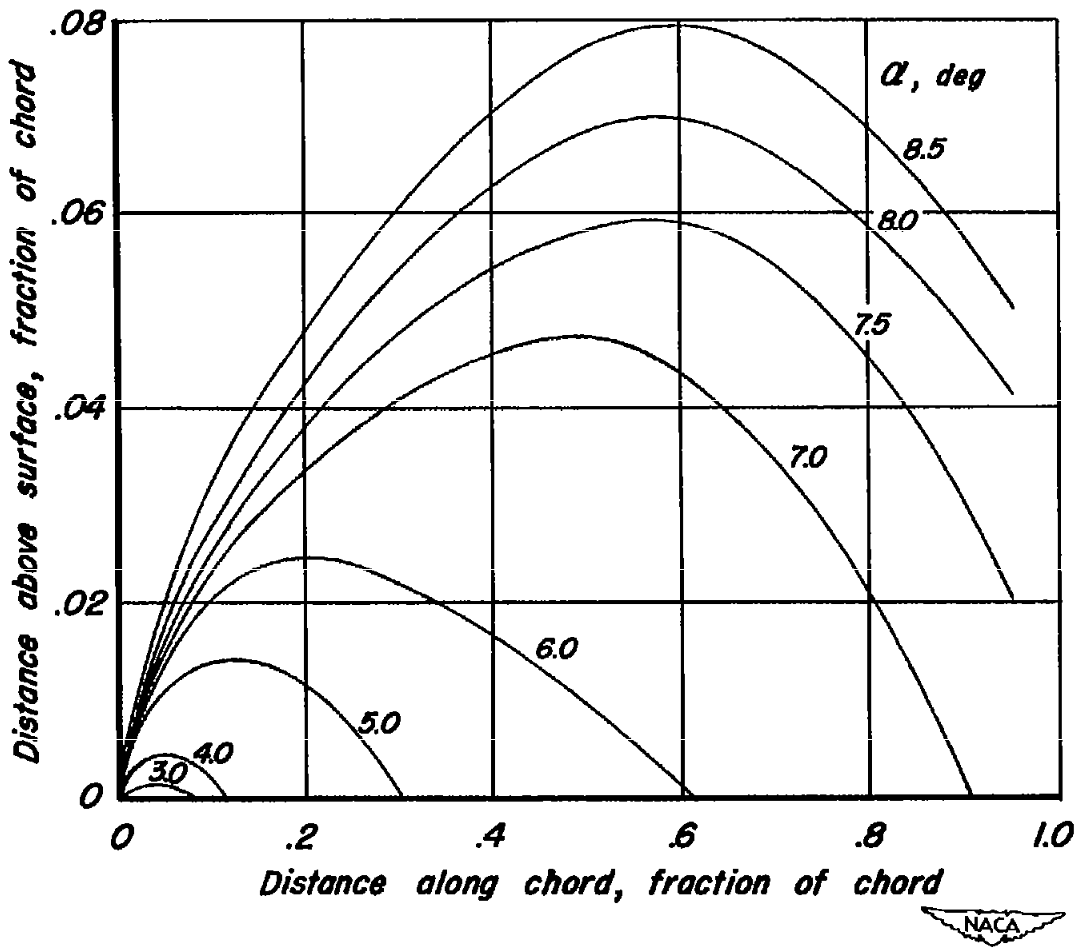


Figure 4.- The extent of upper-surface flow separation on a faired double-wedge airfoil section (ref. 4).

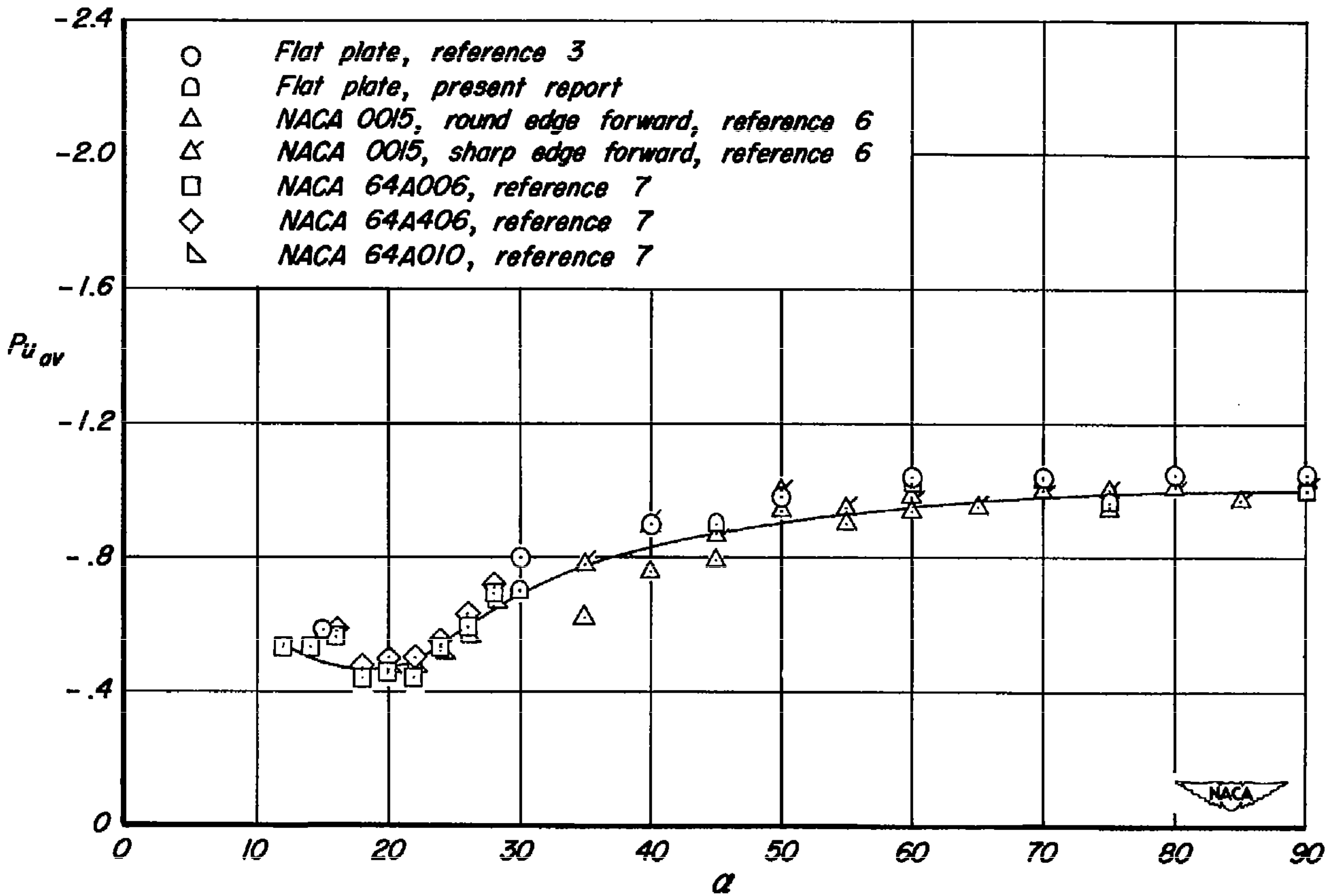
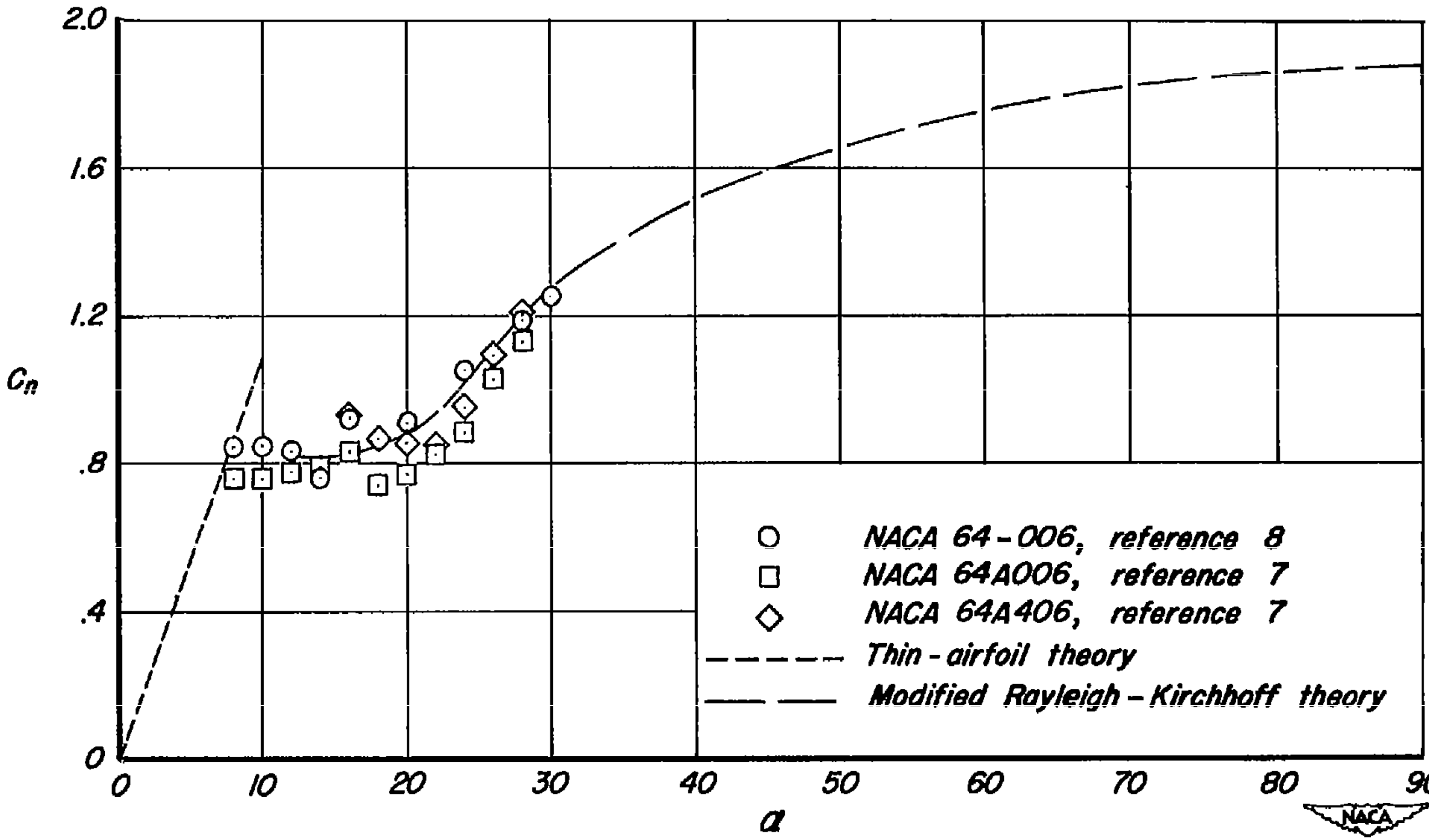
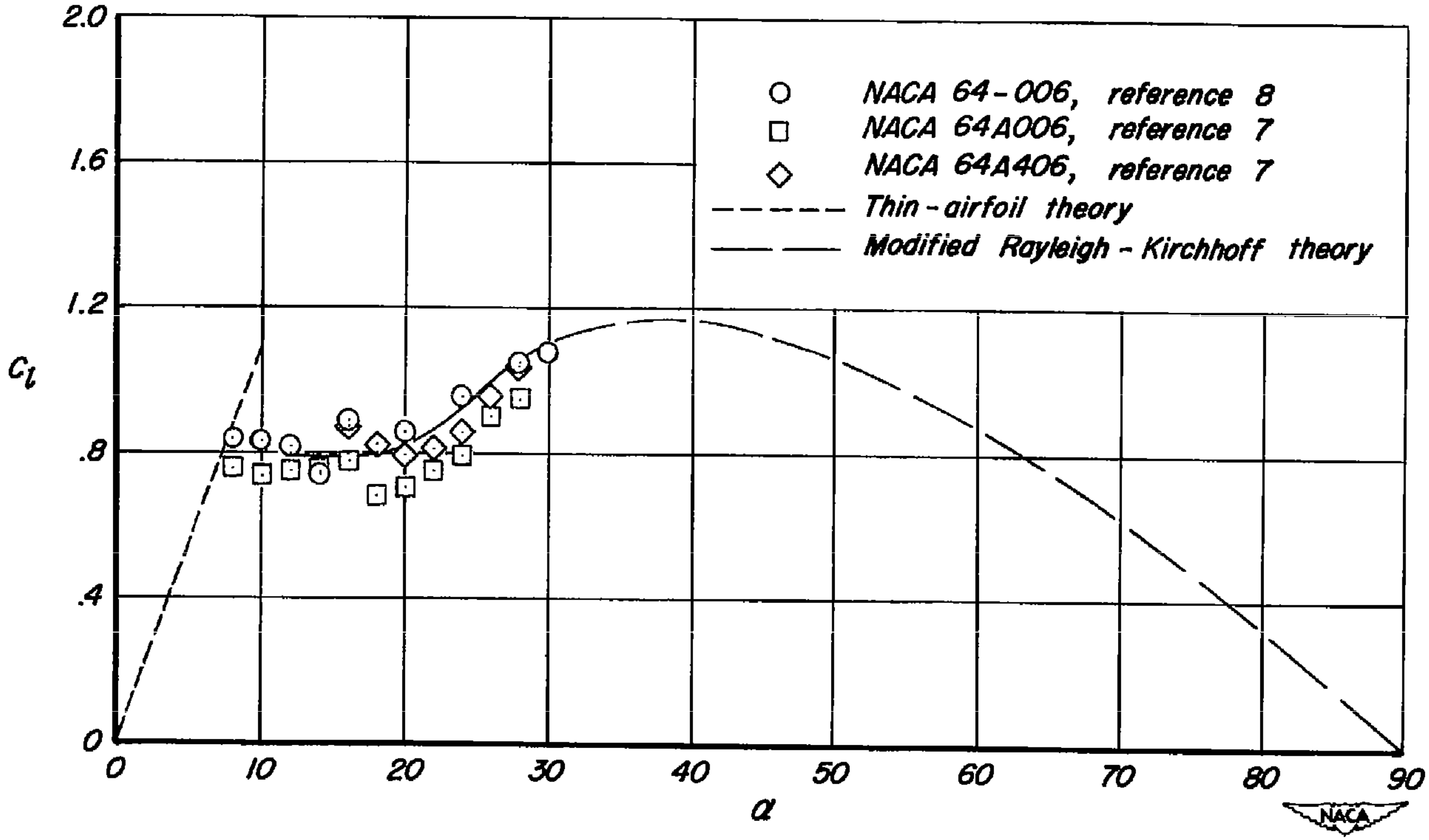


Figure 5.- Average upper-surface pressure coefficients on plates or airfoil sections with completely separated upper-surface flow.



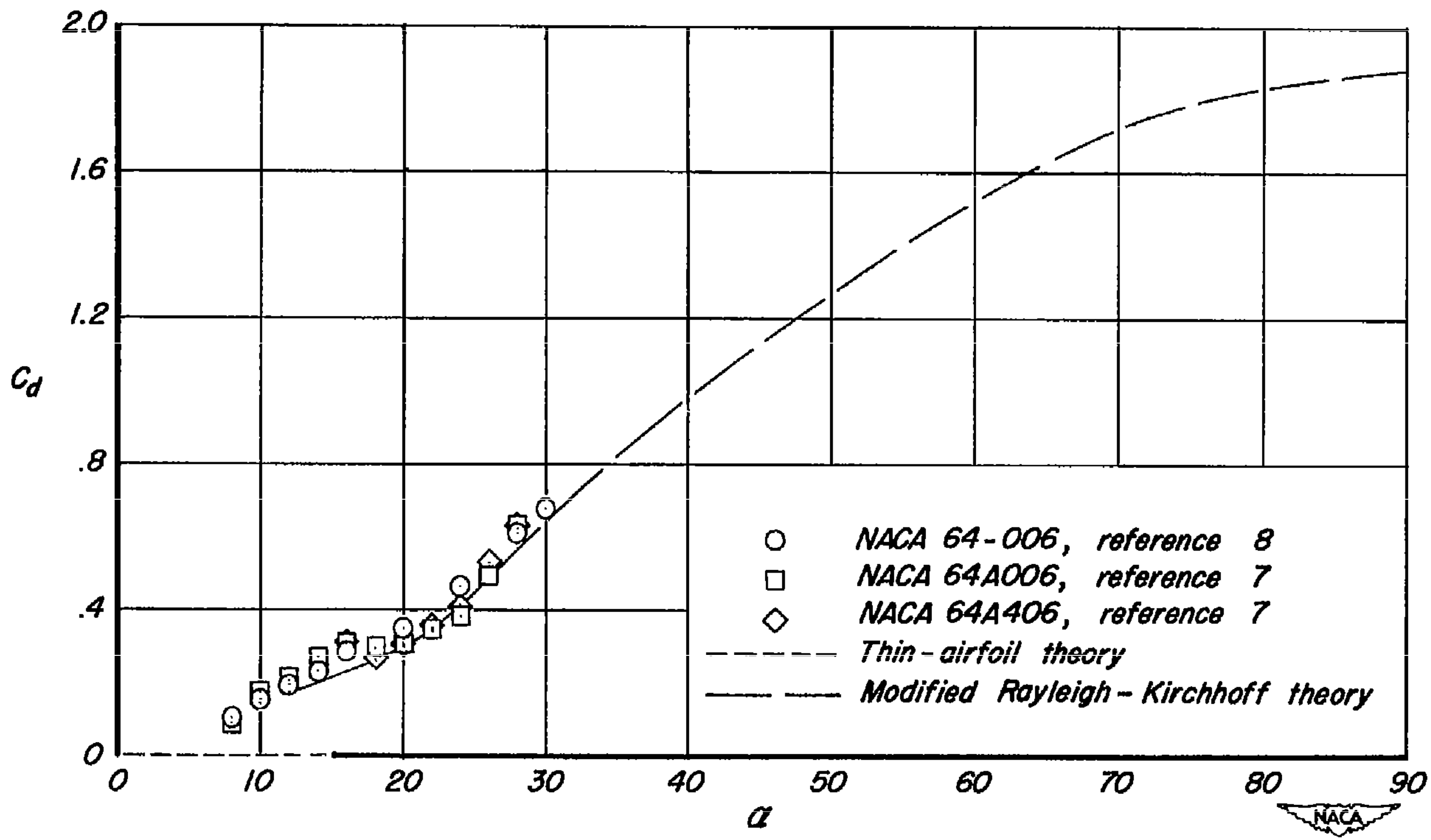
(a) c_n vs. α

Figure 6.- Measured force and moment characteristics for several thin airfoil sections at high angles of attack and theoretical characteristics given by the modified Rayleigh-Kirchhoff theory and thin-airfoil theory.



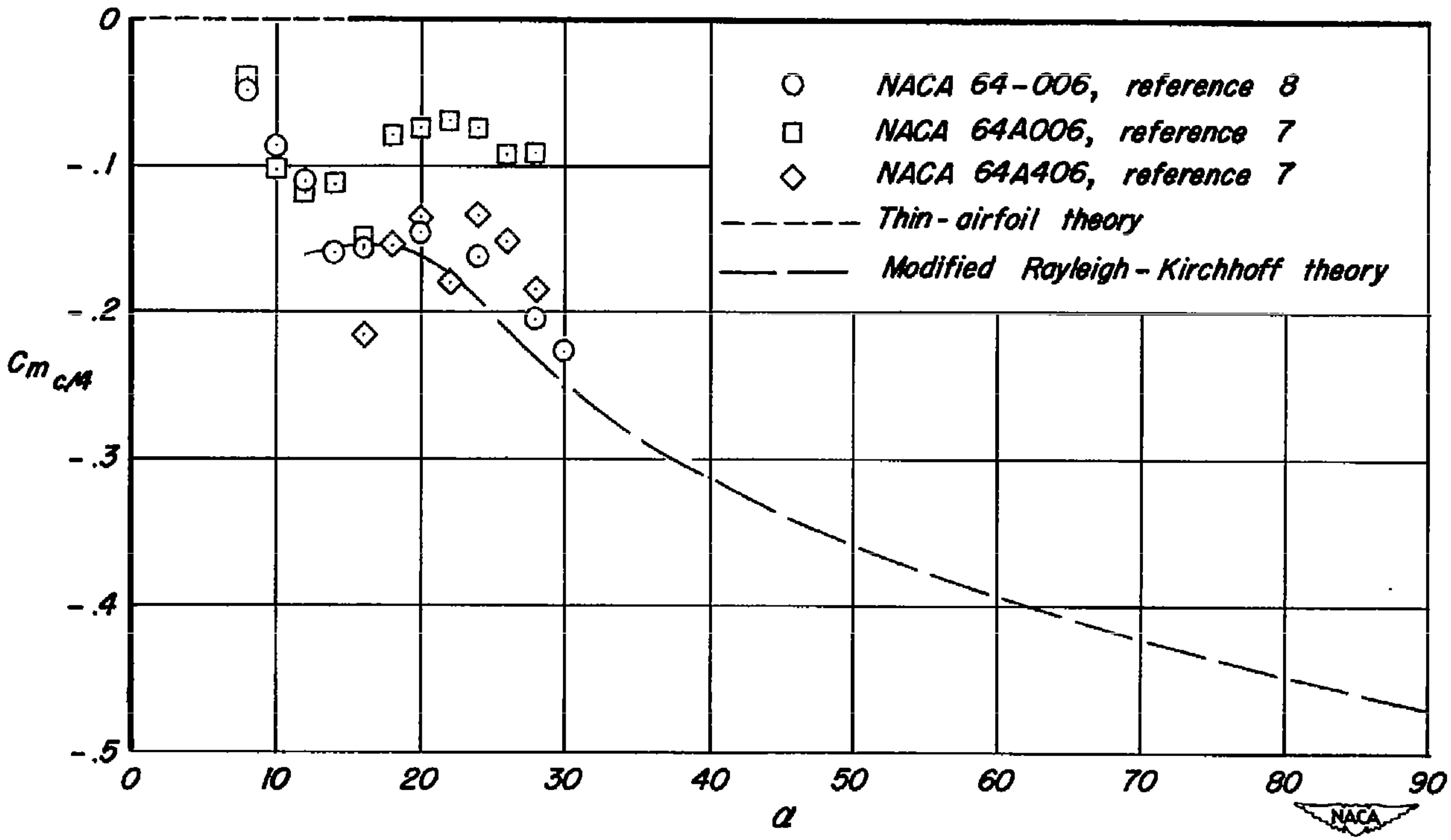
(b) c_l vs. α

Figure 6.- Continued.



(c) c_d vs. α

Figure 6.- Continued.



(d) c_m vs. α

Figure 6.- Concluded.

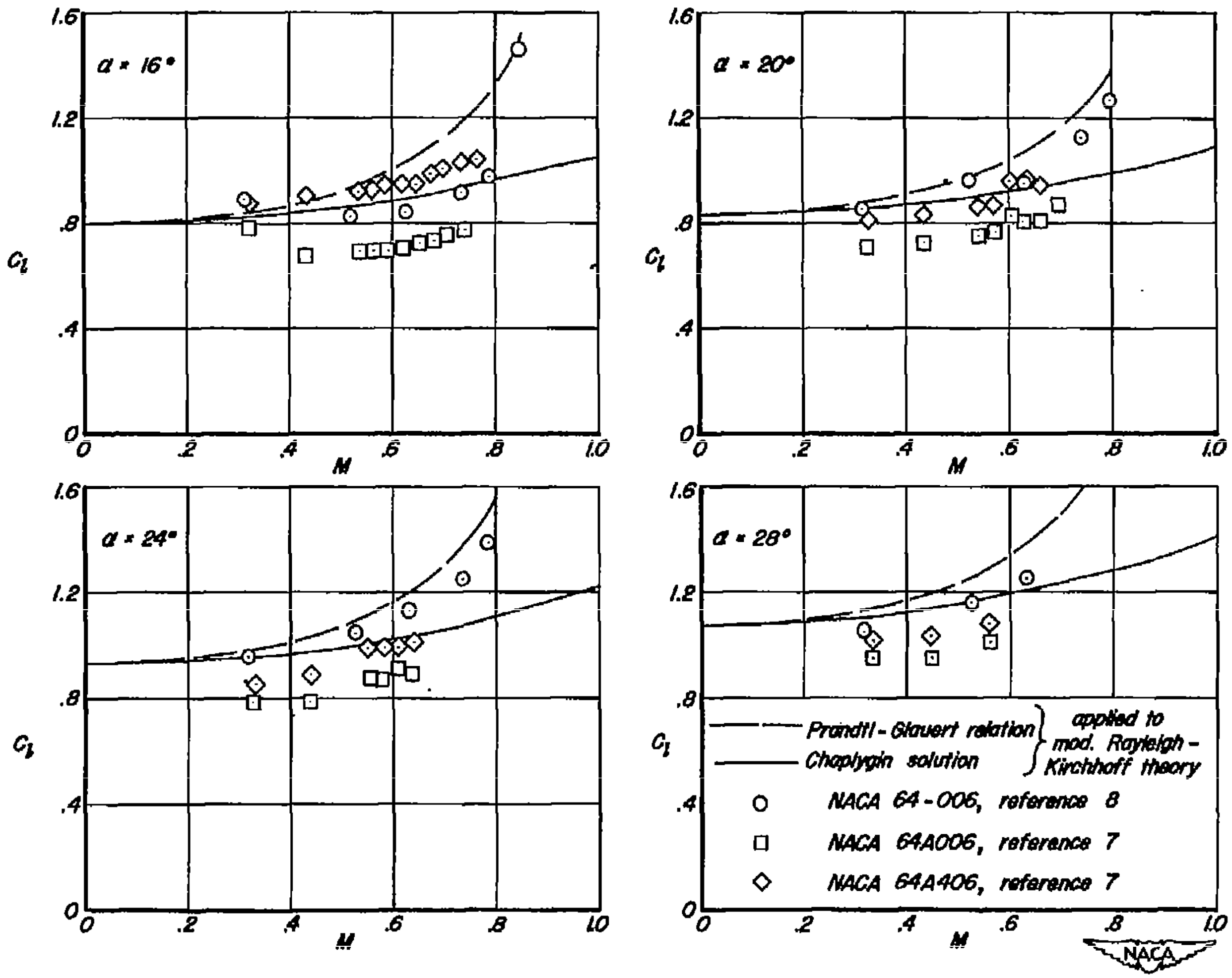


Figure 7.- Comparison of the measured effects of compressibility on the lift characteristics of several thin airfoil sections at high angles of attack with the effects predicted for a flat plate by applying either the Prandtl-Glauert relation or the Chaplygin solution to the modified Rayleigh-Kirchhoff theory.

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